PHAS1102: Physics of the Universe
Problem-Solving Tutorial, Week 15: Notes for Answers

Question 1

The inverse square law tells us that the flux observed from a star at some wavelength (or equivalently, its apparent magnitude) is inversely proportional to the square of its distance: $f \propto d^{-2}$. We also know that a magnitude difference *corresponds to a flux (or brightness)* ratio; *specifically,*

$$m_1 - m_2 = -2.5 \log \frac{f_1}{f_2}$$

By considering these two facts, show that the difference between m, the apparent *magnitude of a star, and M, its* absolute *magnitude, is*

$$m - M = 5 \log d - 5$$

where d is the distance in parsecs.

Imagine some star at a distance of d parsecs, with an apparent magnitude m; label the flux received from that star as $f_{\rm d}$.

Now (as a thought experiment!) we move the star to distance of 10 parsecs. By definition, at this distance its *apparent* magnitude, say m_{10}, equals its *absolute* magnitude, M. Label the flux now received from the star f_{10} (neglecting the possible effects of interstellar dust).

From the inverse-square law we know that

$$\frac{f_{10}}{f_{\rm d}} = \left(\frac{d}{10}\right)^2,$$

and from the relationship between magnitudes and fluxes we know that

$$\begin{aligned} m_{10} - m &= -2.5 \log \frac{f_{10}}{f_{\rm d}}; \qquad \text{i.e.,} \\ M - m &= -2.5 \log \left[(d/10)^2 \right] \\ &= -5 \log d + 5 \log 10, \qquad \text{or} \\ m - M &= 5 \log d - 5 \end{aligned}$$

as required.

(We can easily take account of extinction by interstellar dust, if we measure that extinction in magnitudes.)

Question 2

(A) Accompanying this sheet is a figure showing light-curves of 4 Cepheid variables in the galaxy M100 (a member of the Virgo Cluster), as observed with the Hubble Space Telescope. Magnitudes in the V passband are shown as a function of phase in the pulsation cycle; the pulsation periods (in days) are marked above each frame. Estimate the distance modulus, and the distance (in Mpc), for each Cepheid.

We need to use the given Cepheid P–L relationship to find $M(V)$ from P; and we need to estimate the average $m(V)$ from observations. Differencing gives us the distance modulus directly, and hence the distance. The numbers I get are:

Star	V range	$< m(V) >$	P(d)	$M(V)$	$m - M$	d (Mpc)
C33	25.3–26.5	25.9	31.6	-5.63	31.53	20.2
C35	25.6–26.8	26.2	30.0	-5.57	31.77	22.6
C37	25.8–26.8	26.3	29.7	-5.56	31.86	23.6
C39	25.6–26.6	26.1	28.8	-5.52	31.62	21.1

Students might well estimate mean magnitudes, $< m(V) >$, that are 0.1–0.2m different from mine, but any larger discrepancies should be critically scrutinized. The absolute magnitudes $M(V)$ should match mine.

(B) Are the results from different stars consistent? Can you think of any potential sources of systematic error? Adopt a representative distance for M100, justifying your value.

It's best to think of this in terms of distance moduli, DM (not distances). I think I can estimate $< m(V) >$ to about 0.1m, so the range in my distance moduli, 0.33m, is perhaps a bit more than expected, albeit not alarmingly so.

The obvious source of systematic error is the neglect of interstellar reddening (which students were reminded about in Q1). This will be different for different stars – and its neglect surely contributes to the dispersion in derived DMs.

Students might well pick the average DM as the 'representative distance'. However, given that extinction always makes stars appear too faint (i.e., more distant than they really are), I'd take the smallest value as an upper limit to the true distance to M100: $d \lesssim 20$ Mpc.

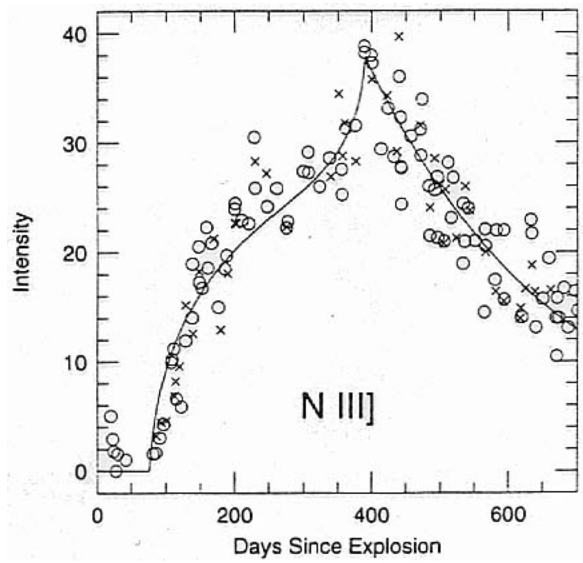
[For info, the value finally adopted by the HST Key Project team was 14.3 Mpc, though they used a slightly different P–L calibration, and considered other distance calibrators.]

(C) The redshift measured for M100 is +1571 km s^{-1}. What value is implied for the Hubble Constant? Is the value reasonable?

Hubble's constant is given by v/d; i.e., for my adopted d, $1571/20 = 78.5$ km s^{-1} Mpc^{-1}. This seems reasonable to me, given that

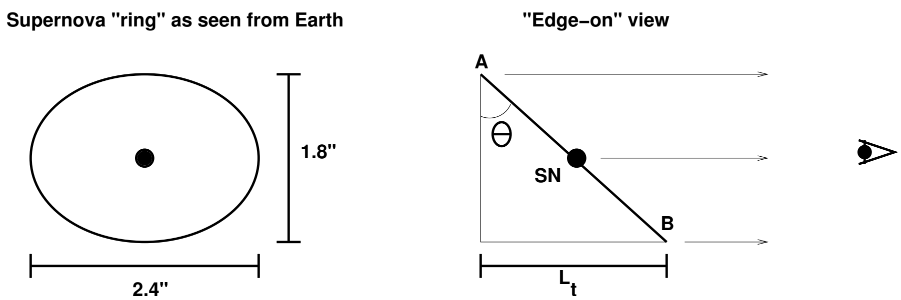
(i) the consensus value of H_0 is about 72 km s^{-1} Mpc^{-1},
(ii) the observed velocity isn't entirely due to the 'Hubble flow', but has a component due to the peculiar velocities of M100 and our own Galaxy.

Question 3



•*Estimate t_0 and $t_{\max}$ from the diagram*
I estimate $t_0 \simeq 80, t_{\max} \simeq 390$. Any reasonable values (say, within 15d of mine) are acceptable.

•*Why is t_0 not zero?*
The astute student will deduce that the time axis in the above diagram is chosen such that zero is the time of observed eruption (in practice, set to the time at which neutrinos from the supernova arrived at Earth).
From the ring geometry,



the time taken for photons to travel from the SN to the Earth is shorter than the time taken for photons to travel from the supernova to the near side of the ring (point ‘B’) to the Earth. Thus we see even the nearest point in the ring ‘light up’ some time after we first see the SN erupt.
[Simple geometry shows that the delay is $R(1 - \sin\theta)/c$ where R is the ring radius.]

• *What is the corresponding value of L_t?*
The answer is just $c \times (t_{\max} - t_0)$, and really only requires attention to units (convert delay in days to delay in seconds before multiplying by c im km/s – or, less conventionally, express c in km/d). For my values of $t_0, t_{\max}$

$$c \times (t_{\max} - t_0) = (3 \times 10^5) \times (310) \times (60 \times 60 \times 24) \qquad = 8.0 \times 10^{12} \text{ km}$$

• *What is the diameter of the ring (in both km and pc)?*
[We're told that the angular length of the major axis of the ring is $a = 1.6$ arcseconds, and that $\theta = 43°$.]

We just have to correct for the projection factor; from the geometry sketched above, the diameter D is just $L_{\rm t}/\sin\theta$, i.e.,

$$\begin{aligned} D &= 8.0 \times 10^{12} \text{ km}/\sin 43° \\ &= 1.2 \times 10^{13} \text{ km} \\ &= 0.38 \text{pc} \end{aligned}$$

• *What is the distance to SN1987A?*
In other words, at what distance d does 0.38 pc subtend an angle of 1.6″?

You can do this with standard trigonometry, or go straight to an answer by noting that for small angles the angular diameter *expressed in radians* is just the linear diameter divided by the distance. My answer (for $\Delta t = 310$d) is 49 kpc.

• After travelling a distance of 49 kpc = 159,817 light-years, neutrinos were observed 19 hours before photons. One interpretation is, therefore, that they travel slightly faster than light, by a fraction

$$\sim \frac{19 \text{ hr}}{159817 \times 365.25 \times 24 \text{ hr}}$$

that is, by a factor 1.36×10^{-8}, or by 4 m/s.

(The recent claim from CERN corresponds to an excess neutrino velocity of about 6 *thousand* m/s!)

Of course, a mismatch in SN photon/neutrino arrival times can easily be explained in other, less exotic ways. An obvious possibility is that it was mostly cloudy in the 24 hours prior to discovery, so the SN simply wasn't noticed. It's also the case that neutrinos travel through the collapsing progenitor more easily than do photons, and so escape the surface first.