PHAS1102: Physics of the Universe
Problem-Solving Tutorial, Week 15

Assume that the Cepheid Period-Luminosity relationship can be expressed as

$$M(V) = -2.80 \log_{10} P - 1.43, \tag{1}$$

where $M(V)$ is the mean absolute magnitude at V (the average of values at minimum and maximum brightness), and P is measured in days.

Question 1
The inverse square law tells us that the flux observed from a star at some wavelength is inversely proportional to the square of its distance: $f \propto d^{-2}$.
We also know that a magnitude *difference* corresponds to a flux *ratio;* specifically,

$$m_1 - m_2 = -2.5 \log \frac{f_1}{f_2}$$

By considering these two facts, show that the difference between m, the *apparent* magnitude of a star, and M, its *absolute* magnitude, is

$$m - M = 5 \log d - 5$$

where d is the distance in parsecs. (For this purpose, ignore any possible dimming of starlight by interstellar dust.) The quantity $(m-M)$ is called the 'distance modulus', and is a way of expressing distance in terms of magnitudes.

Question 2
(A) Accompanying this sheet is a figure showing light-curves of 4 Cepheid variables in the galaxy M100 (a member of the Virgo Cluster), as observed with the Hubble Space Telescope. Magnitudes in the V passband are shown as a function of phase in the pulsation cycle; the pulsation periods (in days) are marked above each frame. Estimate the distance modulus for each Cepheid, and the distance (in Mpc).

(B) Are the results from different stars consistent? Can you think of any potential sources of systematic error? Adopt a representative distance for M100, justifying your value.

(C) The redshift measured for M100 is +1571 km s^{-1}. What value is implied for the Hubble Constant? Is the value reasonable?

Question 3
This question examines how to calculate the distance to Supernova 1987A (and hence the distance to its host galaxy, the Large Magellanic Cloud). The supernova is surround by a circular ring of material, ejected long before the explosion. The ring appears elliptical because it is tilted at an angle θ to our line of sight; θ can easily be estimated directly from the ratio of the minor to major axes of the ellipse.

When the supernova exploded, it emitted a strong burst of ionizing radiation. After 3 months or so, this radiation reached the pre-existing gas ring, causing it to 'light up'; after this, the ring began to fade slowly. Although all parts of the ring were lit at pretty much the same time (because the supernova is at the centre of the circle), we did not *see* all points 'light up' simultaneously, because emission coming from the back side of the ring had to travel further to reach us than emission from the front of the ring (and so arrived later). The interval between the time when light from the front of the ring reached Earth, and light from the back of the ring arrived, represents the light travel time between the near and far sides of the ring. Referring to the accompanying figures (overleaf), the corresponding distance is labelled L_t.

The large lower diagram shows the observed brightness of the ring[1] as a function of time, measured from the moment the supernova erupted. We designate the time at which the ring *starts* to brighten as t_0 (light from the front of the ring first reaches us), and the time when it reached *maximum* brightness as $t_{\max}$ (light from the back of the ring reaches us for the first time). Then

$$L_t = c(t_{\max} - t_0),$$

where c is the speed of light.

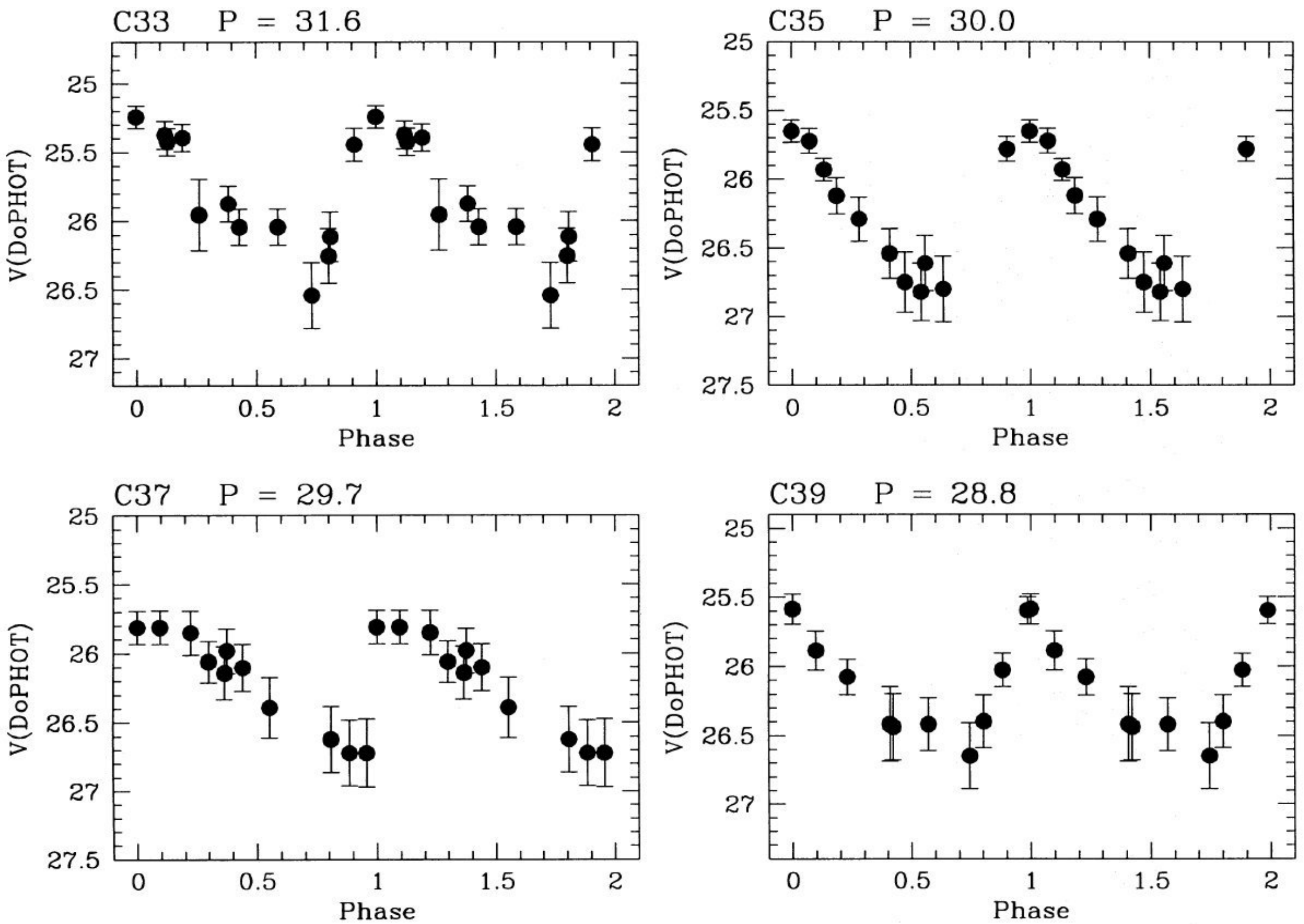
- Estimate t_0 and $t_{\max}$ from the diagram.
  Why is t_0 not zero (in the time units of the diagram)?
  What is the corresponding distance L_t (in km)?

Given that the angular length of the major axis of the ring is $a = 1.6$ arcseconds, and that $\theta = 43^\circ$ (noting that if the ring were in a plane perpendicular to the line of sight, there would be no delay between 'front' and 'back' [indeed, there wouldn't be a 'front ' and 'back']), then

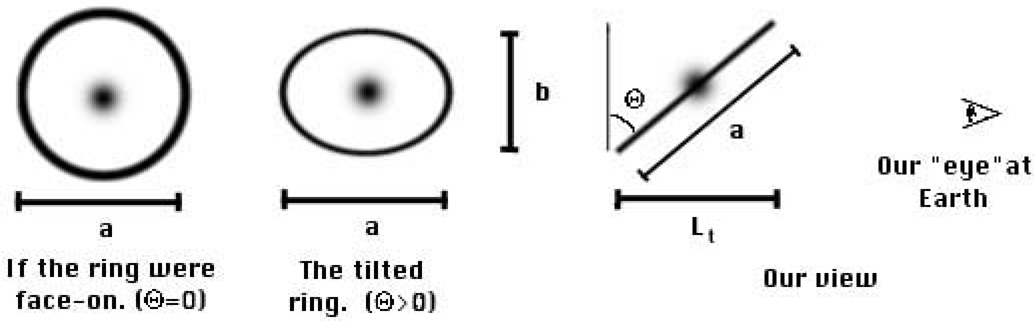
- What is the diameter of the ring? (Express your answer both in km and in pc.)
  What is the distance to SN 1987A?

- Twenty-four (or so) neutrinos were detected from SN 1987A, by three different experiments, over an interval of about 20s on 1987 Feb 23, at 7:36 UT; the supernova was discovered optically at 2:40 UT on Feb 24. Using only your distance estimate and these data, how much faster than light might you conclude that neutrinos travel (express your answer in m/s)? Is faster-than-light neutrino travel the only explanation for the observations?

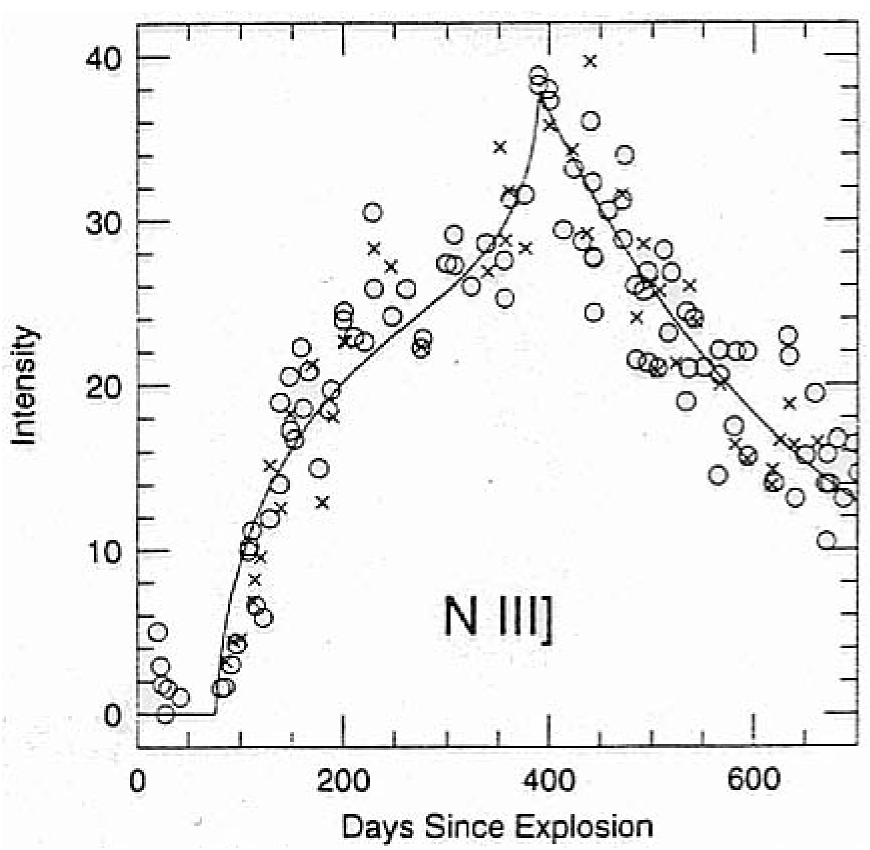
[1]Actually, the intensity of one emission line in the ring's spectrum, but that's an unimportant detail for the present discussion.



Light-curves of 4 Cepheid variables in the galaxy M100 (a member of the Virgo Cluster), as observed with the Hubble Space Telescope.



Geometry of the ring around SN 1987A



Light-curve of the SN 1987A ring