Physics of the Universe
PHAS1102 – Problem Solving Tutorial

You may assume the following:

$$G = 6.67300 \times 10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$$
$$M_\odot = 1.98892 \times 10^{30}\ \mathrm{kg}$$
$$\mathrm{pc} = 3.08568025 \times 10^{16}\ \mathrm{m}$$

and that

$$\int x \exp\left(\frac{-x}{a}\right) \mathrm{d}x = -a(a+x)\exp\left(\frac{-x}{a}\right)$$
$$\frac{mv^2}{r} = \frac{GM(r)m}{r^2}$$

A spiral galaxy is viewed from 'above'. Its *average* luminosity per unit area, as a function of distance r from the centre, is

$$L(r) = L_0 \exp\left(\frac{-r}{r_0}\right)$$

(smoothing out features like the spiral arms). Here r_0 is a 'scale length' – the distance over which the $L(r)$ changes by a factor e.

1. Derive an expression for the total luminosity of the galaxy from the centre to some radius R. (You should start by thinking about the surface area between radii r and $r + \mathrm{d}r$; this should lead you to write down an integral, which you can then evaluate.)

2. For a particular galaxy, the central surface brightness is observed to be

   $$L_0 = 150 L_\odot\ \mathrm{pc^{-2}}$$

   and the scale length is

   $$r_0 = 2\ \mathrm{kpc}.$$

   After making sure you have obtained the right expression for Q1 (check with your class leader), compute the total luminosity (in units of $L_\odot$) within radius R for $R = 0$, 2, 4, 6, 8, 10, 15, 20, and 25 kpc. Make a plot of your results (a rough plot will do).

3. Suppose, as a crude but (superficially) reasonable approximation, that $M(R)/M_\odot$, the mass within radius R, equals $L(R)/L_\odot$, the luminosity within radius R, where both are measured in solar units. Calculate the orbital velocities of stars at the 9 radii used above, and make another rough plot of your results.

4. Suppose that the Sun's distance from the centre of the Galaxy is 8.0 kpc, and its orbital velocity about the centre is 220 km/s. Calculate the mass inside the solar orbit (in units of the solar mass).