Problem Paper II:4

Question 1
The energy density of black-body radiation at temperature T is given by $(4\sigma/c)T^4$ where σ is the Stefan-Boltzmann constant and c is the speed of light. Calculate the energy density of the Cosmic Microwave Background (CMB; assume $T = 2.728$ K).

The typical energy of a photon in a black-body distribution of temperature T is roughly $3kT$, where k is Boltzmann's constant. What is the number density[1] of CMB photons in the Universe today?

[Express all answers to this question in SI units.]

Question 2
In the Week 13 Problem-Solving Tutorial, you were invited to consider a spiral galaxy viewed from 'above'. You should've been able to show that the total luminosity of the disk, integrated from the centre out to some radius R, is given by

$$L(R) = 2\pi L_0 r_0 \left[r_0 - (R + r_0) \exp\left(\frac{-R}{r_0} \right) \right]$$

where L_0 is the central surface brightness and r_0 is the scale length for the particular galaxy. Here we will explore this same topic in more detail.

For a particular galaxy, we suppose the central surface brightness is

$$L_0 = 250 L_\odot \text{ pc}^{-2}$$

and the scale length is

$$r_0 = 1.8 \text{ kpc}.$$

By using a spreadsheet (or other technique of your choice), evaluate and plot the total luminosity (in units of $L_\odot$) for $R =$0–50 kpc at steps of 0.5 kpc.

Suppose, as a crude but superficially reasonable approximation, that $M(R)/M_\odot$, the mass within radius R, equals $L(R)/L_\odot$, the luminosity within radius R, where both are measured in solar units. Plot the predicted rotation curve (the orbital velocities of stars, in km/s, as a function of galactocentric distance, R, in kpc).

Your answer should consist of two plots, plus a table listing $L(R)$, $M(R)$, and $v(R)$ for $R = 0, 10, 20, 30, 40$, and 50 kpc.

[You may find it helpful to look at the notes on PST 3, available on the PHAS 1102 web page.]

[1] 'Number density' just means 'how many per unit volume' (cp. 'mass density', meaning ' how much mass per unit volume'.)