Problem Paper II:1

solar mass	$M_\odot$	1.989×10^{30} kg
parsec	pc	3.0857×10^{16} m
hydrogen mass	m_H	1.66×10^{-27} kg

Question 1
The Schwarzschild radius marks the 'event horizon' of a nonrotating black hole, such as might be found in the centre of a galaxy; classically, we can extract no information from inside this radius, which in some sense represents 'the' radius of a black hole. Although it requires a full relativistic treatment to derive correctly an expression for the Schwarzschild radius, the same result comes out just by finding the radius at which the escape velocity from a body of mass M equals the speed of light. Derive an expression for the Schwarzschild radius in this way (by equating the kinetic energy required to eject a particle at the speed of light with that particle's potential energy).

Question 2
Suppose, for the purposes of a rough calculation, that the Milky Way galaxy contains 10^{11} stars (and nothing else), each weighing one solar mass on average, and each composed entirely of hydrogen (not unreasonable for an order-of-magnitude estimate). Suppose further that the Milky Way is a typical galaxy, and that each cubic megaparsec of space contains 1 galaxy. *On average,* what volume of space is occupied by a single hydrogen atom? (Express your answer in units of cubic metres.)

Question 3
Another rough calculation: if the age of the Universe is 1.4×10^{10} years, estimate the volume of the observable universe. (Remember, this is intended to be a *rough* calculation, so the only extra information you should need to make this estimate is the speed of light. Express your answer in units of cubic megaparsecs.)

Suppose the European Standard Beach is 1km long, 10m across, and 1m deep; suppose also that the European Standard Sandgrain occupies 1 cubic mm. Which is the larger number: the number of stars in the observable universe, or the number of grains of sand on a beach? What about the number of galaxies compared to the number of sand grains? (You will need to re-use some information from Question 2.)