# Problem Paper II:3 – Notes for Answers

**Question 1**

*The velocities of individual galaxies don't obey Hubble's Law exactly – galaxies have 'peculiar velocities' (largely resulting from gravitational interactions with neighbouring galaxies) in addition to 'cosmological velocities' resulting from the Hubble flow.*
*Suppose that a typical peculiar velocity is 500 km s$^{-1}$. How far away would a galaxy have to be if you wanted to use it to determine the Hubble constant with an uncertainty of 10 per cent? (Assume $H_0 = 72\ \mathrm{km\ s^{-1}Mpc^{-1}}$, and that the distance and velocity of the galaxy can be measured perfectly accurately; express your answer in Mpc.)*

The Hubble constant is basically velocity divided by distance, in suitable units (normally km/s and Mpc). This question therefore essentially asks "at what distance does 500 km/s represent 10 per cent of the (cosmological) recession velocity", i.e., to what distance does 5000 km/s correspond? For $H_0 = 72$ km s$^{-1}$ the answer is simply (5000/72) Mpc, or 69 Mpc.

*For the given value of $H_0$, calculate a value for the Hubble time. (Express your answer in years, to 2 significant figures.)*

This question really requires no more than attention to units (plus knowing that there are $3.08568 \times 10^{19}$ km Mpc$^{-1}$, and remembering that the Hubble time, an approximate estimate of the age of the universe, is just $1/H_0$):

$$\begin{aligned}
\tau_0 &= \frac{1}{H_0} \\
&= \frac{3.08568 \times 10^{19}\ \text{km Mpc}^{-1}}{72\ \text{km s}^{-1}\ \text{Mpc}^{-1}} \\
&= 4.29 \times 10^{17}\ \text{s} = 1.4 \times 10^{10}\ \text{yr}
\end{aligned}$$

**Question 2**

*The velocity of recession of a galaxy is very often expressed in terms of its redshift, $z$. If $v \ll c$ we can convert from redshift to velocity by using the usual Doppler formula,*

$$1 + z \equiv \frac{\lambda}{\lambda_o} \simeq 1 + \frac{v}{c} \quad \text{(i.e., } v \simeq cz\text{);}$$

*otherwise the relativistic form must be used:*

$$1 + z \equiv \frac{\lambda}{\lambda_o} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}.$$

*To show that $v$ and $cz$ are significantly different even for quite small redshifts, rearrange the special-relativistic formula for $z$ into an equation for $v/c$.*

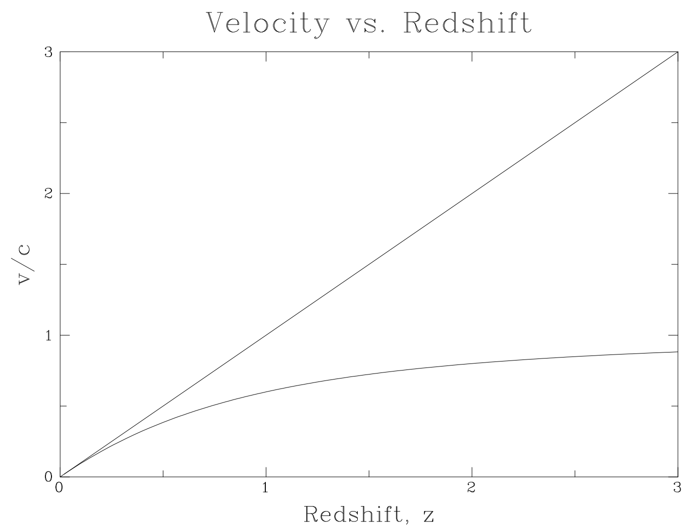
*[*Hint: *to obtain an expression for $v/c$, try writing $(1 - v^2/c^2)$ as $(1 - v/c)(1 + v/c)$.]*

Following the 'hint', we write

$$\begin{aligned} 1 + z &\equiv \frac{\lambda}{\lambda_o} \\ &= \frac{1 + v/c}{\sqrt{(1 - v/c)(1 + v/c)}} \\ &= \sqrt{\frac{(1 + v/c)(1 + v/c)}{(1 - v/c)(1 + v/c)}}; \\ (1 + z)^2(1 - v/c) &= (1 + v/c) \\ \frac{v}{c} &= \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1} \end{aligned}$$

*Plot a graph of $v/c$ vs. $z$ for $z$ in the range 0 to 3; for comparison, also plot the line for the (unrealistic, hypothetical, non-relativistic) case $v = cz$. Put redshift on the $x$ axis.*

Evaluating this over the required range gives a plot like this:



Note that the relativistic and non-relativistic lines diverge substantially for redshifts in excess of just a few tenths; and that by redshift 3, galaxies are receding at about 88% the speed of light (and not $3c$!).