Problem Paper II:2 – Notes for Answers

Question 1

• *Suppose that the Sun's distance from the centre of the Galaxy is 8.0 kpc, and its orbital velocity about the centre is 220 km/s. Calculate the length of the Galactic 'year' (the time taken for the Sun to complete one orbit around the Galaxy), expressing your answer in (Earth) years.*

The circumference C of the orbit (reasonably assumed to be circular) is $2\pi r$; that is, in km,

$$C = 2\pi \times 8.0 \times 3.0857 \times 10^{16}$$
$$= 1.551 \times 10^{18} \text{ km}$$

so the Galactic 'year', in years, is distance/velocity, or

$$C/v = (1.551 \times 10^{18} \text{ km})/(220 \text{ km s}^{-1})$$
$$= 7.050 \times 10^{15} \text{ s}$$
$$= 7.050 \times 10^{15}/(365.25 \times 24 \times 60 \times 60) \text{ yr}$$
$$= 2.23 \times 10^{8} \text{ yr}$$

• *Calculate how many times the Sun has orbited the Galaxy. (Assume that the age of the Sun is* 0.5×10^{10} *yr.)*

$$0.5 \times 10^{10}/2.23 \times 10^{8} \simeq 22(.4) \text{ times.}$$

• *Calculate the total mass contained within the solar orbit, expressing your answer in solar masses.*

All problems of this sort rely on assuming a stable orbit, in which the inward gravitational force is in balance with the outward centrifugal force, i.e.,

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

where, in this case, M is the mass contained inside the orbit, and m, v, and r are the mass, orbital velocity, and orbital radius of the Sun. Rearranging,

$$M = \frac{v^2 r}{G}$$
$$= (220^2 \times 8.0 \times 3.0857 \times 10^{16} \text{ km}^3 \text{ s}^{-2})/G$$
$$= (1.195 \times 10^{22} \text{ km}^3 \text{ s}^{-2})/G$$

we take $G = 6.67300 \times 10^{-11}$ m^3 kg^{-1} s^{-2},

$$= 6.67300 \times 10^{-20} \text{ km}^3 \text{ kg}^{-1} \text{ s}^{-2},$$

whence

$$M = 1.790 \times 10^{41} \text{ kg}$$
$$= 8.9 \times 10^{10} M_\odot$$

Question 2

•*Using Ned Wright's 'cosmology calculator' (with default values for cosmological parameters) tabulate the distances of galaxies at redshifts of 0.1, 0.5, 1.0, 2.0, 4.0, 6.0, 10.0 and 100.0, together with their ages (the time since the Big Bang).*

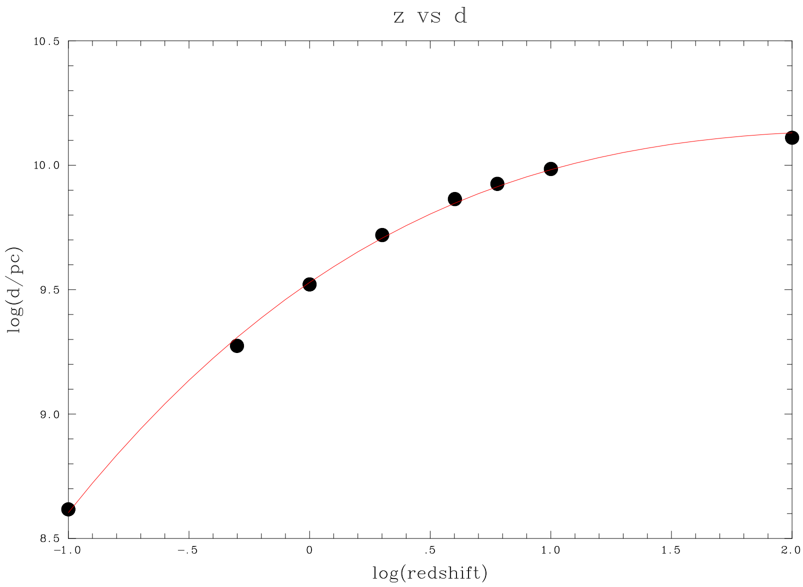
I find:

z	d	age
0.1	414 Mpc	12.38 Gyr
0.5	1.88 Gpc	8.65 Gyr
1.0	3.32 Gpc	5.94 Gyr
2.0	5.24 Gpc	3.34 Gyr
4.0	7.31 Gpc	1.57 Gyr
6.0	8.42 Gpc	0.95 Gyr
10.0	9.66 Gpc	0.48 Gyr
100.0	12.9 Gpc	16.8 Myr

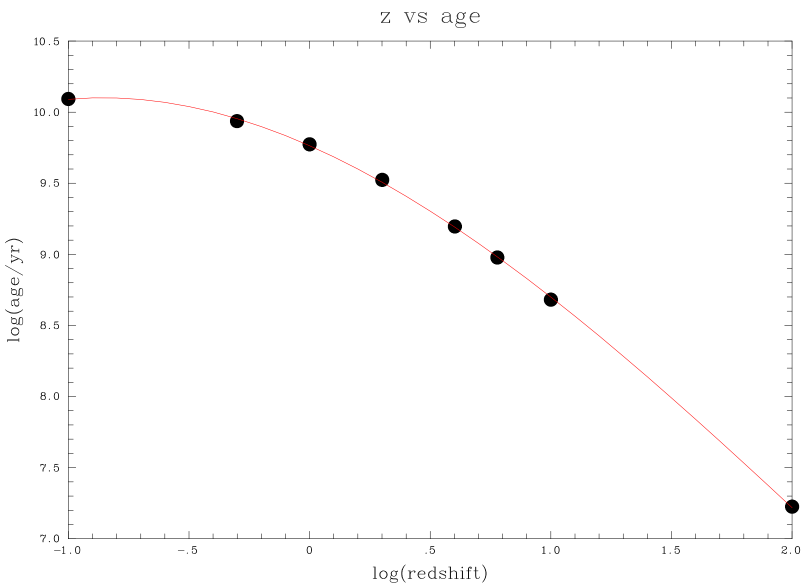
where d is the comoving radial distance.

You'll see that the size of the 'observable universe' that comes from Wright's detailed calculation, a bit more than 13 Gpc, is bigger than our previous simple estimate in Problem Paper II:1 (by about a factor of 3, for the cosmological parameters that go into the detailed calculation). It's also interesting that looking back to 'only' $z = 1$ already takes us back to a time when the universe was less than half its present age.

Students were invited to plot results graphically, as an optional exercise. Overleaf is what I get (for a flat universe, and comoving radial distances).



Observed distance as a function of redshift



Observed age as a function of redshift