Physics of the Universe
In-Course Assessment, 2011 Dec 14

Assume the following values where necessary:

Speed of light	c	3.0×10^{8} m s^{-1}
Gravitational constant	G	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k	1.381×10^{-23} J K^{-1}
Stefan-Boltzmann constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
hydrogen mass	$m_{\rm H}$	1.66×10^{-27} kg
Hubble constant	H_0	72 km s^{-1} Mpc^{-1}
solar mass	$M_\odot$	1.989×10^{30} kg
parsec	pc	3.0857×10^{16} m
year	yr	365.25 d

All answers should be given in SI units unless indicated otherwise.

Question 1: Sketch the rotation curve of a spiral galaxy.

Question 2: The Virgo Cluster of galaxies is 16.5 Mpc distant. What do you expect its velocity of recession to be (in km s^{-1})?

Question 3: What is the Hubble time (in years)?

Question 4: Suppose that the Sun's distance from the centre of the Galaxy is 8.5 kpc, and its orbital velocity about the centre is 250 km/s. What is the mass of the Galaxy contained within the Sun's orbit? (Give your answer in kg, and in units of the solar mass.)

Question 5: A star is observed to orbit the black hole at the centre of our Galaxy. The orbit is circular, with an orbital period of 15.2 years and an orbital radius of 5.5 light-days.
Determine the distance travelled by the star in one orbit (in metres), and hence its orbital velocity.
Use your results to estimate the mass of the Galactic-centre black hole, expressing your answer in units of the solar mass.

Question 6: The energy density of black-body radiation at temperature T is given by $(4\sigma/c)T^4$ where σ is the Stefan-Boltzmann constant and c is the speed of light. Calculate the energy density of the Cosmic Microwave Background (CMB; assume $T = 2.73$ K), in SI units of J m^{-3}.

The typical energy of a photon in a black-body distribution of temperature T is roughly $3kT$, where k is Boltzmann's constant. What is the number density of CMB photons in the Universe today?

Suppose that the average number density of hydrogen atoms in the local universe is one per cubic metre. Express this as an energy density ($E = mc^2$). Compare (i) the number densities and (ii) the energy densities of hydrogen atoms and CMB photons.

[Turn over]

Question 7: The Friedmann equation describes the dynamical evolution of the Universe; it is given by

$$\dot{R}^2 = \frac{8\pi G\rho(t)R^2(t)}{3} - kc^2 + \frac{\Lambda}{3}R^2(t)$$

where $R(t)$, $\rho(t)$ are the 'scale factor' and the mean density of the universe at time t; k is the curvature term; and Λ is the cosmological constant.

If $\Lambda = 0$, space has a flat geometry ($k = 0$) if the mass density equals the 'critical density', $\rho_{\rm c}$.
Re-arrange the Friedmann equation into a form giving the critical density for a $k = 0$, $\Lambda = 0$ universe.
Thereby derive the present-day value for $\rho_{\rm c}$ (in kg m^{-3}).
How many hydrogen atoms per cubic metre are required to match this critical density?

(Recall that the Hubble constant $H_0 \equiv \dot{R}/R$.)