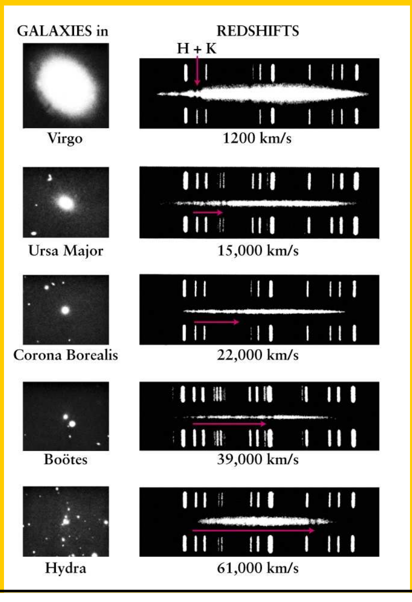
PHAS1102, Section 2 part 3

The Observational Basis of Modern Cosmology

Edwin Hubble



Velocity is used as a surrogate for distance –
km/s for v<<c, otherwise redshift, *z*, where

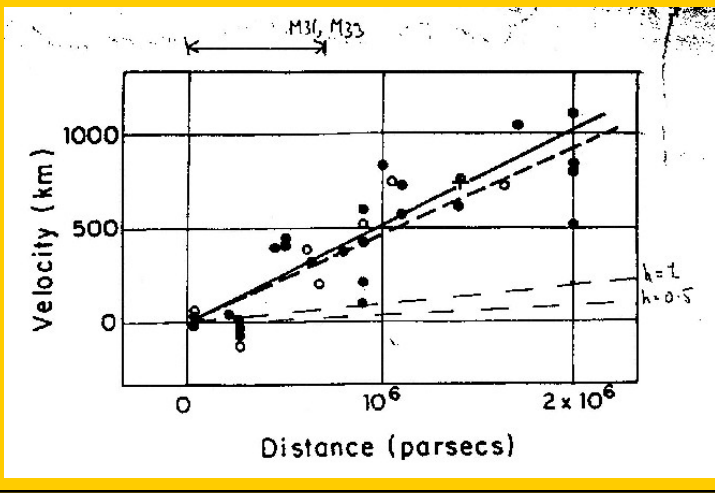
$$1 + z \equiv \frac{\lambda}{\lambda_0} = \frac{\lambda_0 + \Delta\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0}$$

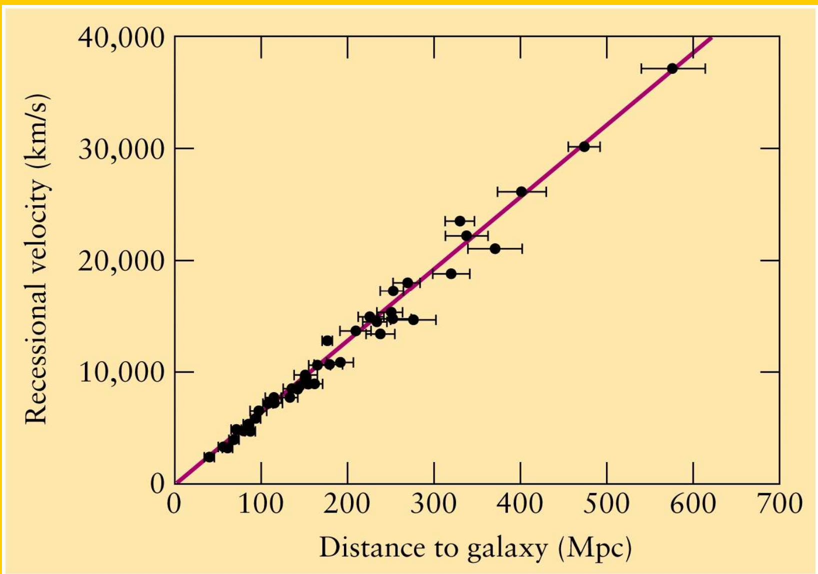
$$1 + z = \frac{\lambda}{\lambda_o} \simeq 1 + \frac{v}{c}$$

$$1 + z = \frac{\lambda}{\lambda_o} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}}.$$

Hubble's Law (published 1929): $v = H_0 d$
$H_0 = 100h$

Expansion allows a rough estimate of the age of the Universe:
$t = d/v = 1/H_0$





H_0 = 72 (+/-5) km/s per Mpc

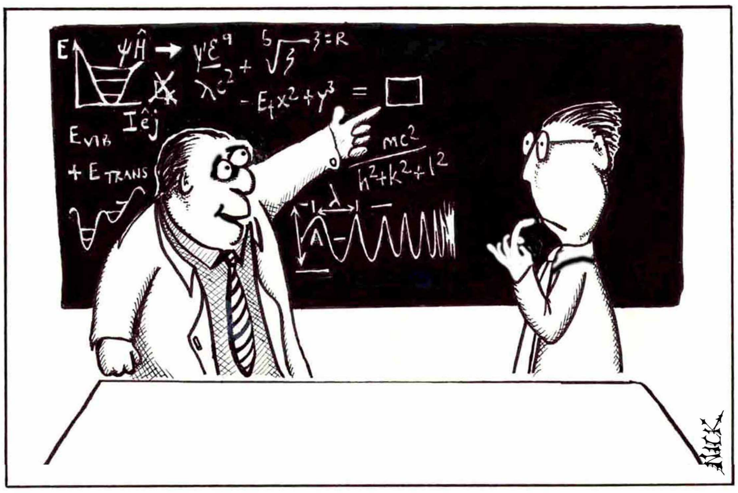
N.B. Everything appears to be moving away from us, but in reality everything is moving away from everything (on cosmological scales; not true locally... Also, expansion of the universe does not mean expansion of the *contents* of the universe.)

→'Big Bang' model

Convenient BUT WRONG to think in terms of a point explosion:

The universe is (almost certainly) infinite, and always has been infinite, even at the time of the big bang....(brain hurt time). 'Size of the universe' generally means 'what we can see'

Cartoon Physicists develop a grand unified theory:



"If my calculations are correct, not only must we always wear white lab coats, but the boundaries of our existence are defined solely by what is allowed to occur within the confines of a small two-dimensional box..!"

'Big Bang' model (*model*, not a theory) is in accord with:

•Hubble Flow (expansion of the universe)

•Evolution of source counts

•Cosmic Microwave Background (CMB)

•Primordial abundances of elements

(Other, more detailed, features are consistent with inflationary and 'λCDM' *theories*)

Evolution of source counts; *n* sources per unit volume, each of intrinsic brightness *L*.

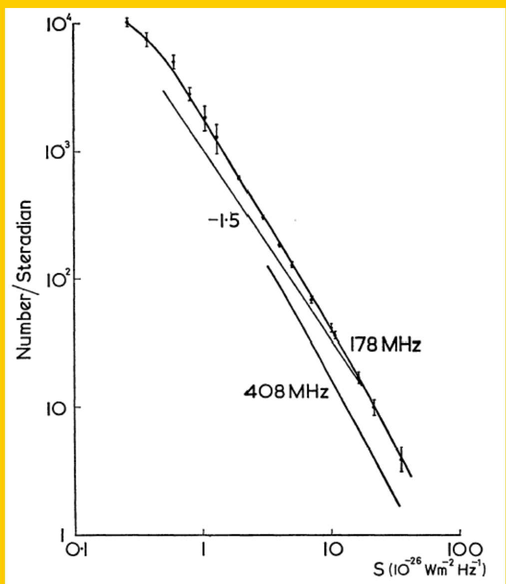
Number of sources per steradian out to distance *r:*

$$N(r) = \frac{4\pi r^3}{3} . \frac{n}{4\pi} = \frac{nr^3}{3}$$

$$\ell \propto L/r^2 \text{ (i.e., } r^2 \propto L/\ell).$$

$$N(\ell) \propto \frac{nL^{3/2}}{\ell^{3/2}}$$

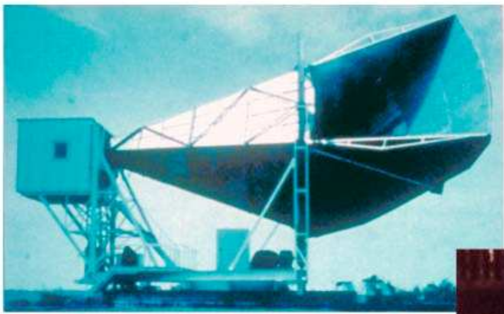
Evolution of source counts (Martin Ryle; 1974, first Nobel Prize for astronomical research)



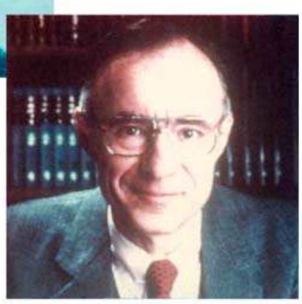
More objects than expected with increasing volume, i.e., with increasing distance, i.e., with decreasing age –

there were more radio sources per unit volume in the past than there are now

DISCOVERY OF COSMIC BACKGROUND



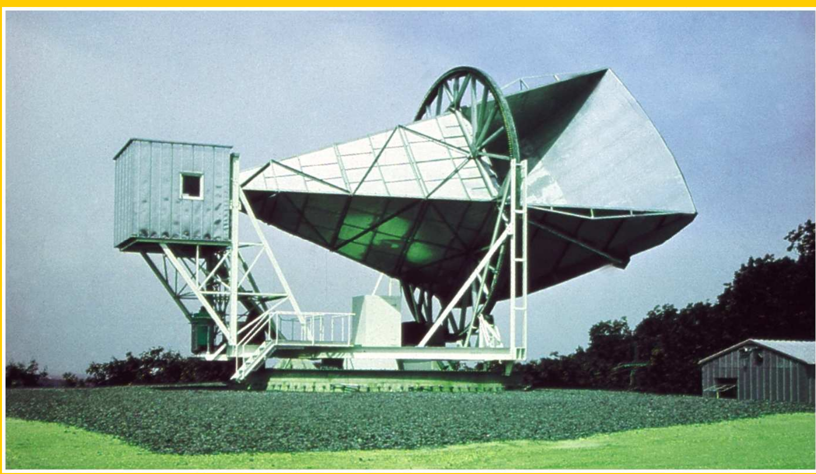
Microwave Receiver



Arno Penzias

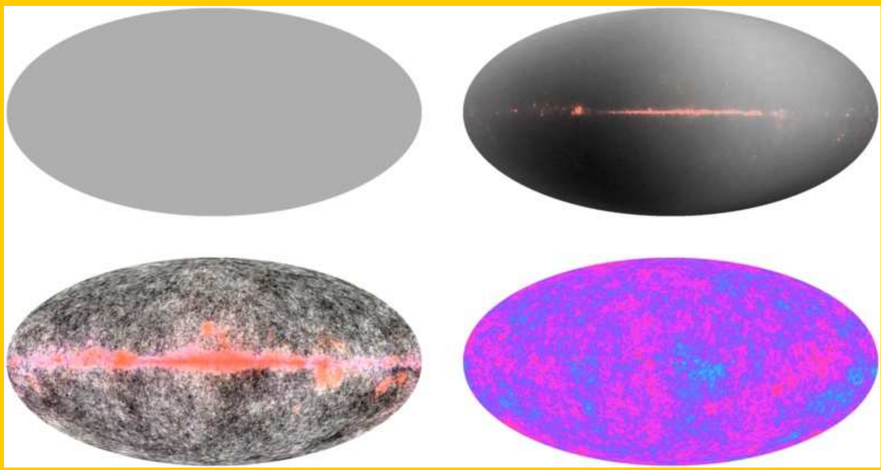
Robert Wilson

MAP990045



The observed Cosmic Microwave Background (CMB):
all-sky images

(Scaled from 1 part in 1000 to 1 part in 100,000)



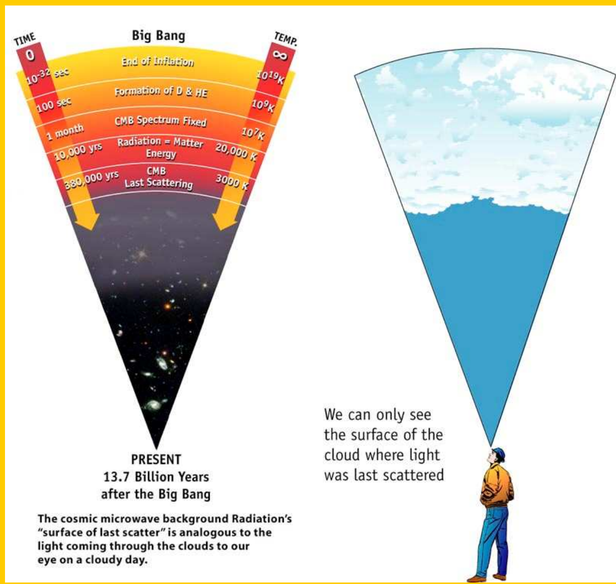
Relic radiation: the Cosmic Microwave Background (CMB)

Initially, the universe was fully ionized, and opaque (or 'optically thick', because of electron scattering).

Matter and radiation strongly interacted → black-body distribution of radiation

Eventually, the universe cooled sufficiently for (hydrogen) atoms to form (T = 3000K, t = 100,000yr, z = 1000).

Radiation decoupled from matter, and has flowed through the universe ever since. We see this 'last scattering surface' redshifted, and it appears to us as a black body at T = 3K (or 2.7K, or 2.725K, depending on how picky you are)



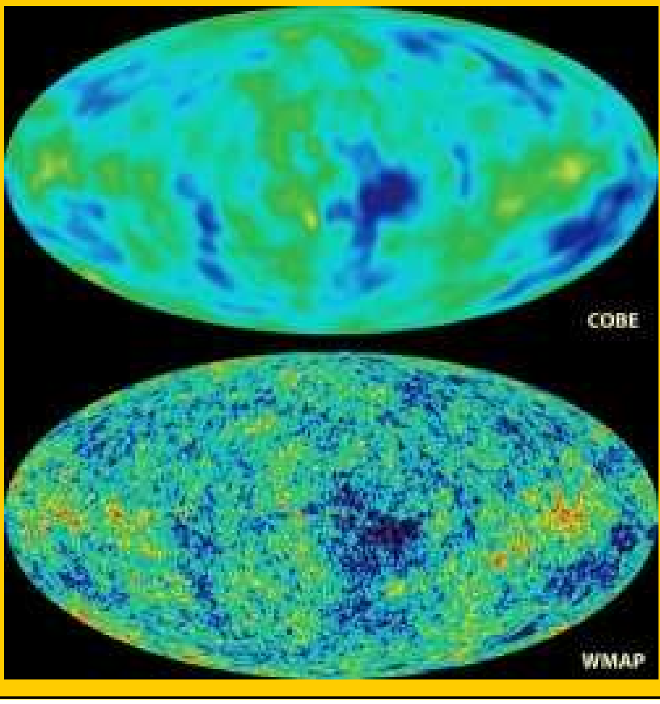
As the early universe cooled, the initially ionized hydrogen would 're'combine with free electrons. We might expect this to occur roughly when

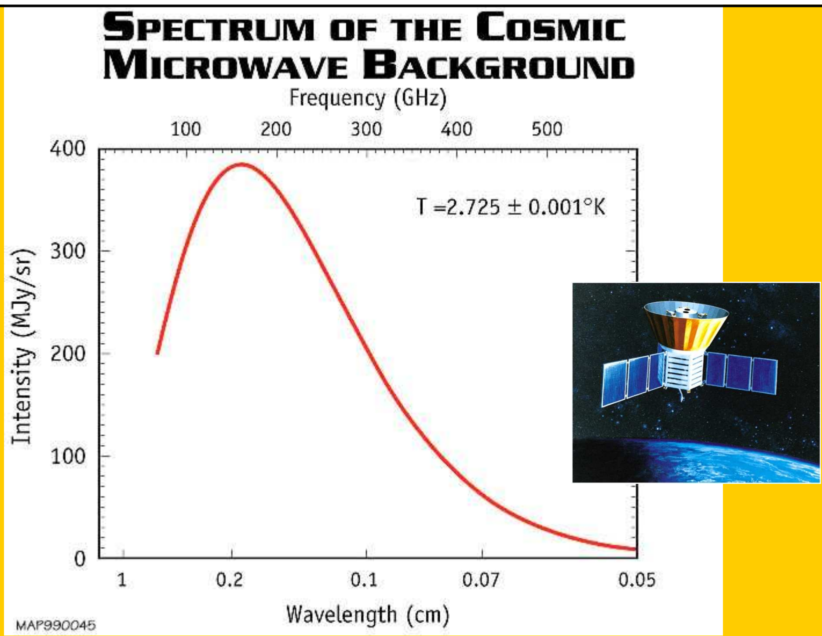
$$kT \sim 13.6\text{eV} \rightarrow T \sim 1.5 \times 10^5\text{K}.$$

However, photons outnumber electrons by a factor $\sim 10^9$ (then, and now).[3] Also, there is a distribution of photon energies at a given temperature (described by the Planck function). In consequence there were still plenty of photons capable of ionizing hydrogen, even at temperatures well below 10^5K, and a better estimate of the temperature at which the radiation field just failed to ionize hydrogen is

$$\sim (13.6\text{eV}/k)/\ln(10^9) \simeq 7600\text{K}.$$

More detailed calculations show that the hydrogen first recombined at $T \sim 3000$K.





COBE spectrum: the most precise black body in nature

PRIMORDIAL NUCLEOSYNTHESIS

- Protons are lighter than neutrons (938.3MeV vs. 939.6MeV; $\Delta m = 1.3$MeV)
- Free neutrons decay ($n \to p + e^-$), with a half-life of $\sim$940s
- Bound neutrons (in nuclei) are stable

At high temperatures the numbers of neutrons and protons are practically identical (since the mass-energy difference between them is negligible compared to kT):

$$n \leftrightarrow p + e^- + \overline{\nu}_e(+0.8\text{MeV})$$

(plus other processes). However, when $kT \sim 0.8$MeV neutrons can convert to protons, but not vice versa. Thus protons begin to outnumber neutrons, by a factor

$$\frac{n_p}{n_n} \sim \exp\left(\frac{\Delta m}{kT}\right) \sim \exp\left(\frac{1.3}{0.8}\right) \sim 5$$

The protons and neutrons combine to build up complex nuclei. Densities are too low at this stage for many-body collisions to be important, so nuclei build up through chains of two-body collisions, starting with $p + n \to^2 D + \gamma$ and then proceeding through a variety of routes, schematically as follows:

$$^2\text{D} + n \to ^3\text{T} + \gamma \quad ^2\text{D} + p \to ^3\text{He} + \gamma \quad ^2\text{D} + ^2\text{D} \to ^3\text{He} + n \quad ^2\text{D} + ^2\text{D} \to ^4\textbf{He} + \gamma$$

$$^3\text{He} + n \to ^4\textbf{He} + \gamma$$

$$^3\text{He} + n \to ^3\textbf{T} + p$$

$$^3\text{T} + p \to ^4\text{He} + \gamma$$

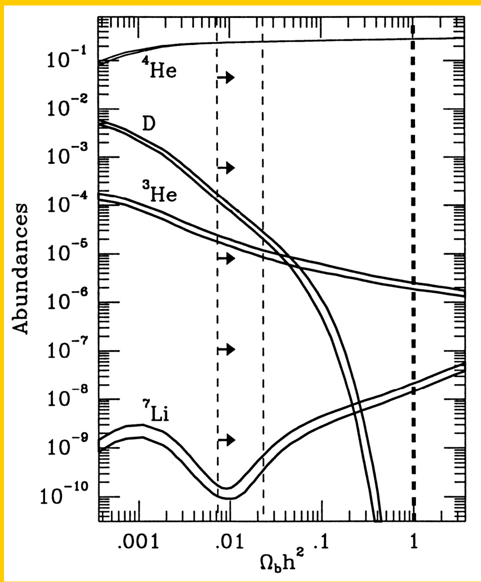
The key stage is the production (and destruction) of deuterium, which has a low binding energy ($\sim$2.2MeV), and so is destroyed at $T \geq 10^9$K. Nucleosynthesis actually occurs at around $kT \sim 0.1$MeV ($T \sim 3 \times 10^8$K), for a period around $t \sim 400$s. This timescale is long enough for the decay of free neutrons to be significant, but not complete; these decays reduce the neutron:proton number ratio by a further factor of $\sim \exp(400/940)$ to give a final neutron:proton ratio of

$$n_n/n_p \simeq 1/7.$$

The only elements produced in bulk are ^{1}H and ^{4}He (the only stable low-mass nuclei), with virtually all the neutrons in helium. Thus the number density of helium atoms is $n(^4\text{He}) = n_n/2$; each weighs 4 neutron masses (to a very good approximation) so the mass density is $2n_n m_H$. The total mass density is $m_H(n_n + n_p)$, so the mass fraction of helium is

$$Y_4 = \frac{2n_n}{n_n + n_p} = \frac{2}{1 + n_p/n_n} \simeq \frac{2}{1+7} = 0.25$$

COSMIC ABUNDANCES



In its early phases (but not too early! – about 100s after 'the start') conditions throughout the universe were not too dissimilar to stellar interiors → primordial nucleosynthesis (creation of H, He etc.)

Particular importance: constrains baryon content of the universe:

$\Omega_B = 0.04$

[but $\Omega_M = 0.27$; *Most matter is 'dark'!*]

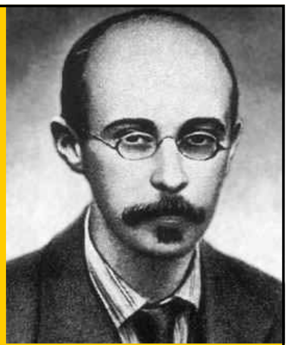
DYNAMICS OF THE UNIVERSE

After the big bang, we might expect the rate of expansion to slow down under the influence of gravity. The rate of deceleration is expected to depend on how much mass there is.

The mass content is conventionally expressed as Ω_M, a fraction of the mass required to bring the universe to a halt after infinite time.

DYNAMICS OF THE UNIVERSE

The Friedmann Equation describes this:



$$\dot{R}^2(t) = \frac{8\pi G\rho R^2(t)}{3} - kc^2 + \frac{\Lambda}{3}R^2(t).$$

R is the ‘scale factor’
Λ is the ‘cosmological constant’ term
k is the ‘curvature’: -1, 0, +1 = negative, flat, positive geometry

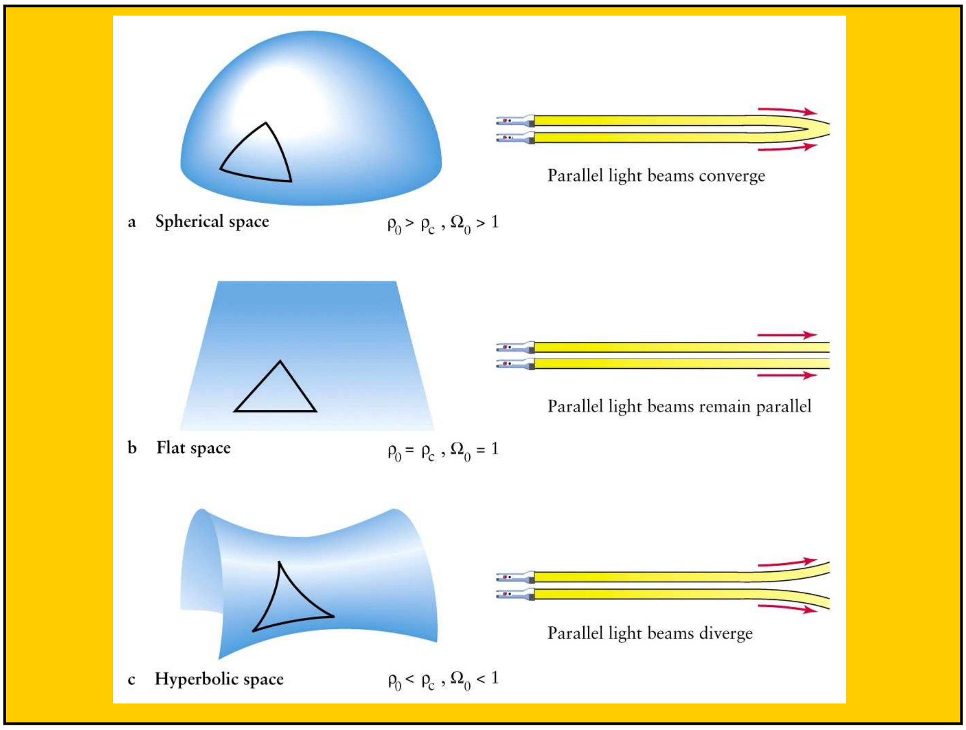
Curvature has a simple but limited ***dynamical interpretation*** **<u>IF</u>** *only gravity matters; then positive geometry corresponds to a closed universe (recollapses), negative geometry corresponds to an open universe (expands forever)*

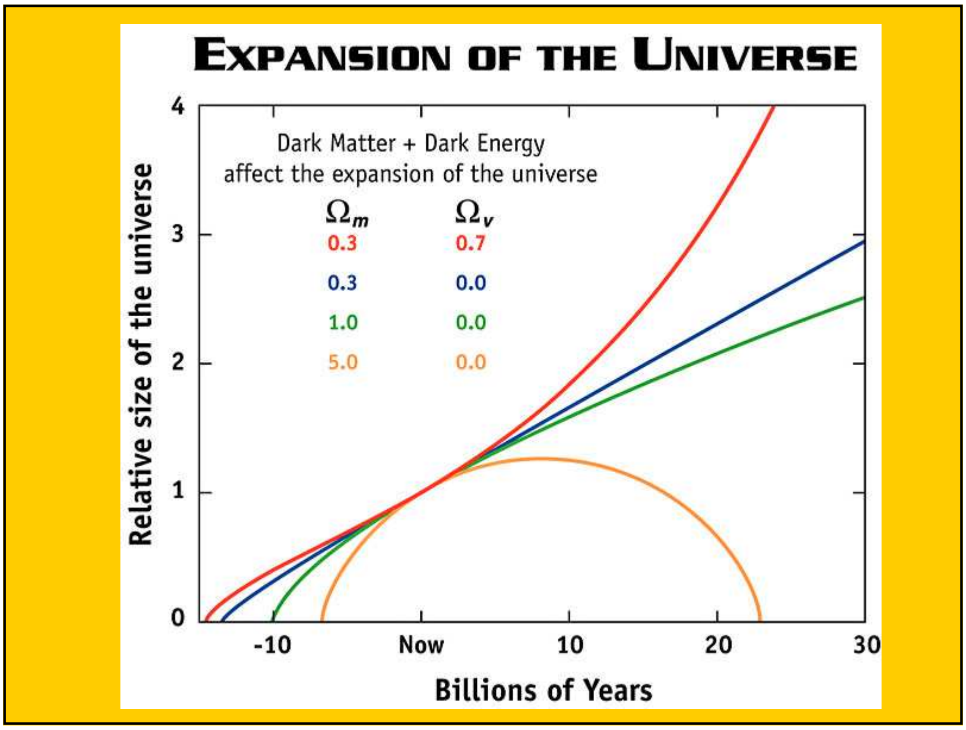
$\Omega_M > 1$ – matter will stop universal expansion after finite time (followed by recollapse, and a ‘Big Crunch’); a closed universe
<u>Geometry</u>: positive curvature, $k = +1$

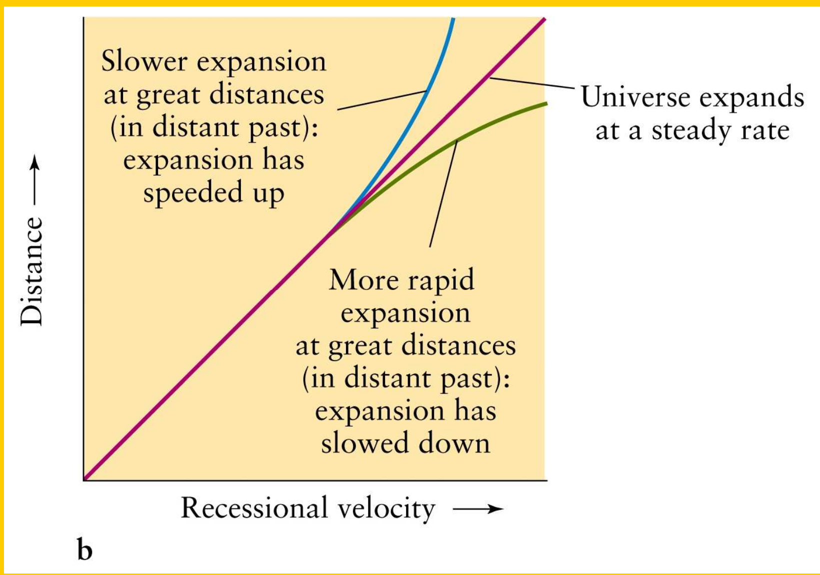
$\Omega_M < 1$ – gravity will never stop the expansion (an open universe)
<u>Geometry</u>: negative curvature, $k = -1$

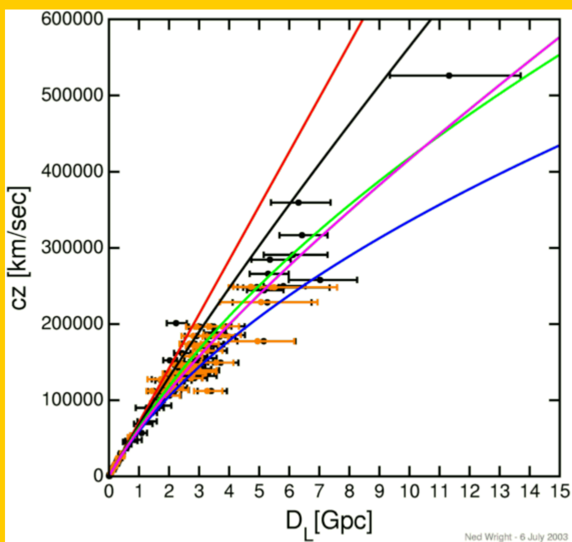
$\Omega_M = 1$ – a critical universe
<u>Geometry</u>: flat (Euclidean), $k = 0$

(*The simple relationship between Ω_M and geometry* **<u>only</u>** *applies if matter is the only important ingredient in determing the dynamics of the universe*)


Parallel light beams converge
a Spherical space
$\rho_0 > \rho_c$, $\Omega_0 > 1$
Parallel light beams remain parallel
b Flat space
$\rho_0 = \rho_c$, $\Omega_0 = 1$
Parallel light beams diverge
c Hyperbolic space
$\rho_0 < \rho_c$, $\Omega_0 < 1$

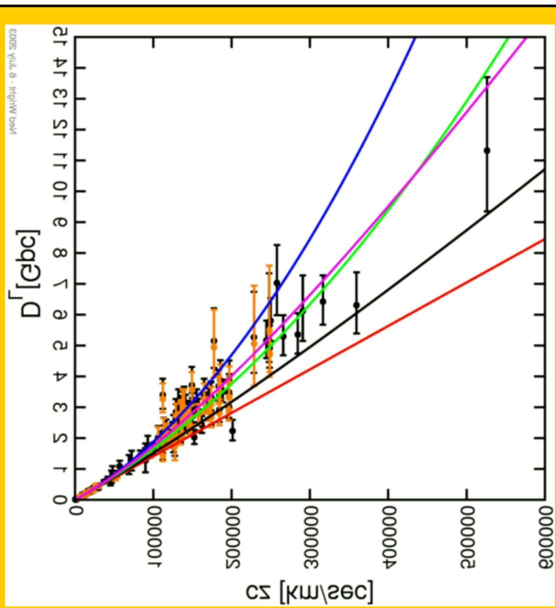

Expansion of the Universe
Dark Matter + Dark Energy
affect the expansion of the universe
Ω_m Ω_v
0.3 0.7
0.3 0.0
1.0 0.0
5.0 0.0
Relative size of the universe
0
1
2
3
4
-10
Now
10
20
30
Billions of Years





SUPERNOVA COSMOLOGY

Distance →



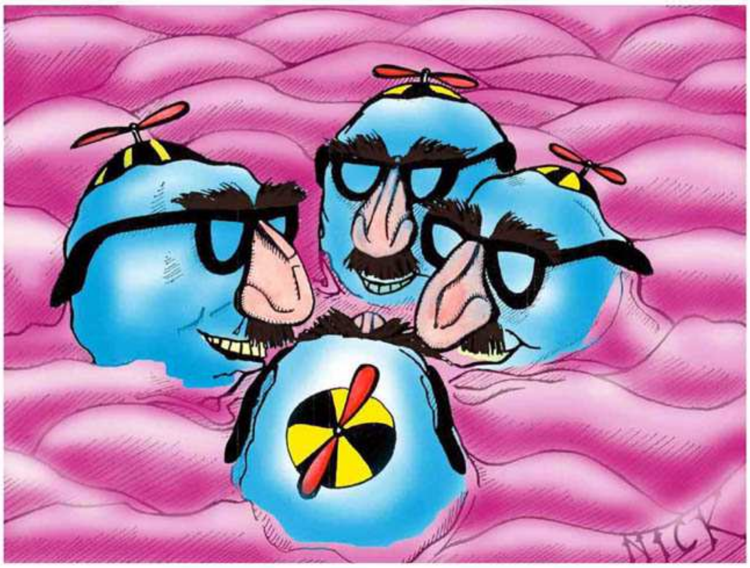
Velocity →

SUPERNOVA COSMOLOGY

In seeking to determine the deceleration of the universe,
it turns out that it's not decelerating at all;

the universe is accelerating!

What can cause this? 'Dark Energy' (aka vacuum energy, cosmological constant, quintessence…

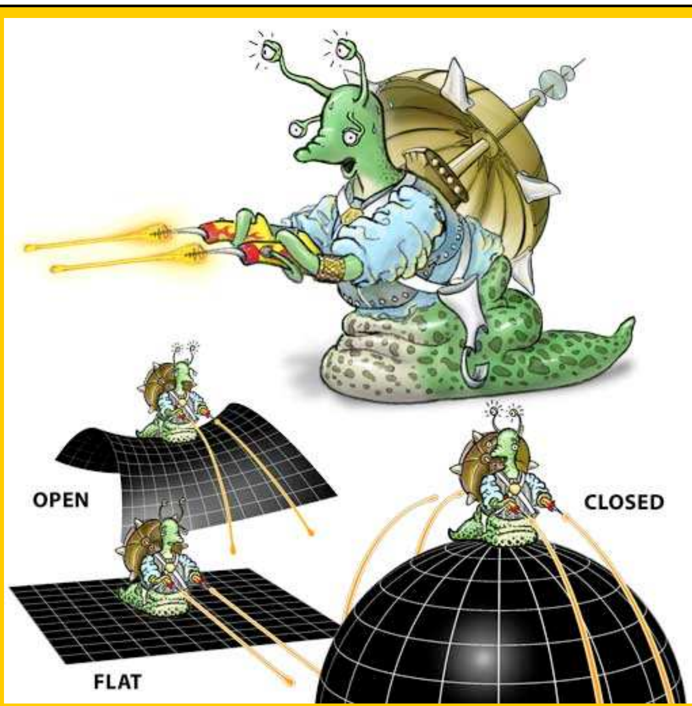


At a resolution of 10^{-24} metres, isolated clumps of Strange Matter pop briefly out of the quantum foam to debate the possible existence of Particle Physicists.

We generalize Ω to include not just matter, but all forms of mass/energy:
$\Omega_M = \Omega_B + \Omega_{DM}$ (+ Ω_ν....)

$\Omega = \Omega_M + \Omega_\Lambda$ (+ $\Omega_{starlight}$, Ω_{CMB}...)

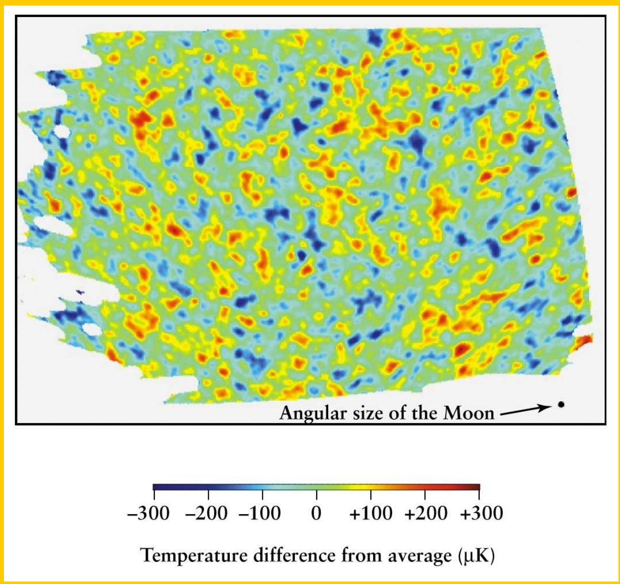
What is the geometry (i.e., what is Ω_{total}?)



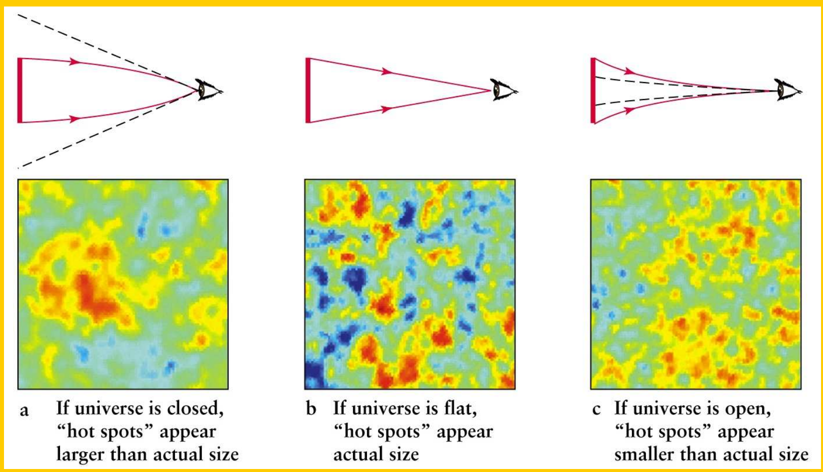
Q: How do we determine geometry?

A: From small-scale structure in the CMB

Balloon Observations Of Millimetric Extragalactic Radiation ANd Geophysics



CMB is constant to 1 part in 100,000....but does have structure



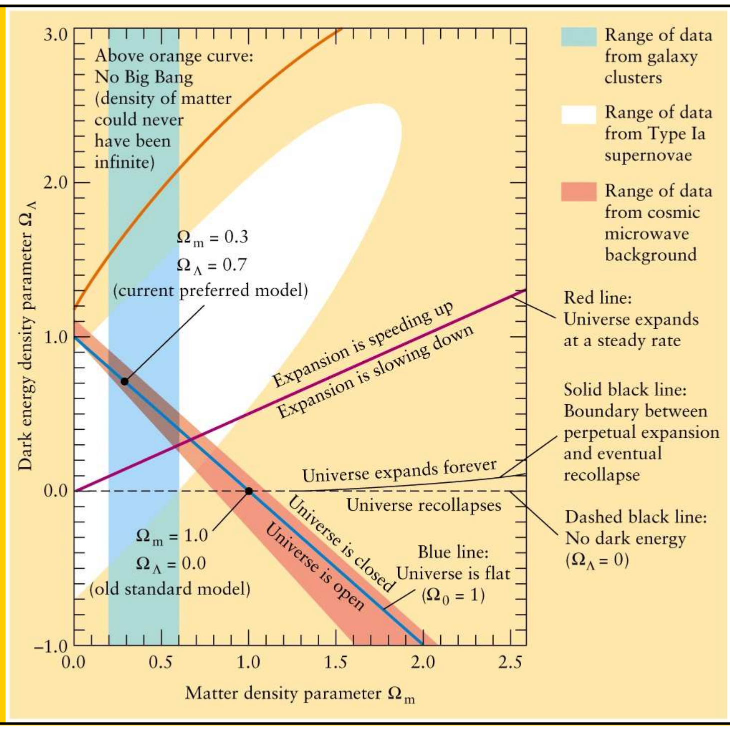
The geometry of the Universe is *flat* (Ω_{Total} = 1) to within a few per cent (implies very close to flat)

But observationally, Ω_B = 0.04, and Ω_M = 0.27

What makes up the rest? Not starlight, not neutrinos, not CMB but *dark energy* (aka the cosmological constant/vacuum energy) Ω_Λ = 0.7

(corresponds to an equivalent mass density of the vacuum of 10^{-26} kg m^{-3} – 120 orders of magnitude smaller than particle-physics estimate!)

→ 'Concordance Model'



CONCORDANCE COSMOLOGY:

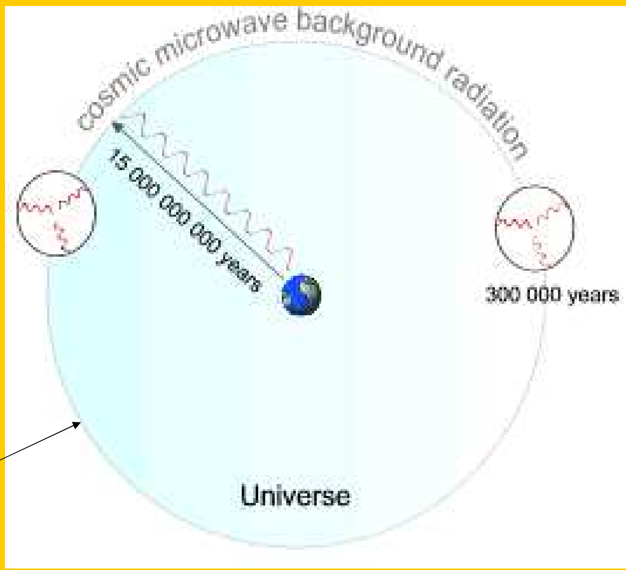
Description	Symbol	Value	+ uncertainty	- uncertainty
Total density	Ω_{tot}	1.02	0.02	0.02
Equation of state of quintessence	w	< - 0.78	95% CL	--
Dark energy density	Ω_Λ	0.73	0.04	0.04
Baryon density	$\Omega_b h^2$	0.0224	0.0009	0.0009
Baryon density	Ω_b	0.044	0.004	0.004
Baryon density (cm^{-3})	n_b	2.5×10^{-7}	0.1×10^{-7}	0.1×10^{-7}
Matter density	$\Omega_m h^2$	0.135	0.008	0.009
Matter density	Ω_m	0.27	0.04	0.04
Light neutrino density	$\Omega_\nu h^2$	< 0.0076	95% CL	--
CMB temperature (K) [a]	T_{cmb}	2.725	0.002	0.002
CMB photon density (cm^{-3}) [b]	n_γ	410.4	0.9	0.9
Baryon-to-photon ratio	η	6.1×10^{-10}	0.3×10^{-10}	0.2×10^{-10}
Baryon-to-matter ratio	$\Omega_b \Omega_m^{-1}$	0.17	0.01	0.01

H_0 = 72 km/s/Mpc Ω_M=0.27 Ω_Λ=0.73 (Ω_{Tot} = 1.00)
Age of the Universe: $1.37.10^{10}$ yr

Problems with the Big Bang:

- It's a model
The *Horizon Problem
The *Flatness Problem
+
Structure Problem
Monopole Problem

Solution: 'Inflation'



Flatness problem: *why* is Ω so close to 1?

Even if you don't believe it's 1, it's certainly within a factor 10 of 1 – which means it must have been really close to 1 previously.

Structure problem: If we take a typical length scale of 1Mpc in the present-day universe, and project back to t~10^{-30} s, the length scale is ca. 0.1mm

This is small, but still much larger than the Hubble scale at that time – ca. 10^{-23} mm!

So how did large-scale structure come about? (Another sort of 'horizon problem')

Monopole problem: where are they??

Why is the vacuum force repulsive?

Consider a piston filled with (false) vacuum; pull out the piston to increase the volume by an amount V. The false-vacuum mass increases by an amount $\rho_{vac}V$, and its energy by $\rho_{vac}Vc^2$, which must correspond to the work done on the piston, $-pV$. Thus the false vacuum has a *negative* pressure, $p = -\rho_{vac}c^2$.

["Equation of State": $w = p/\rho$
w = -1 for Cosmo Const, otherwise "quintessence";
for w<-1, "big rip".]

A negative pressure doesn't sound promising for inflating the universe, but it isn't pressure itself that's important (even in familiar circumstances, only pressure *differences* do).

<u>But....</u>

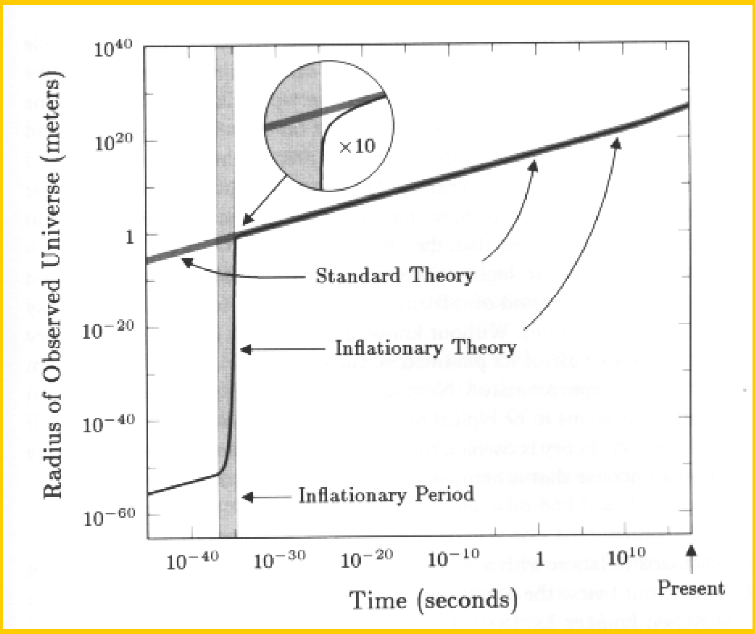
In GR, pressure has associated with it a gravitational field!

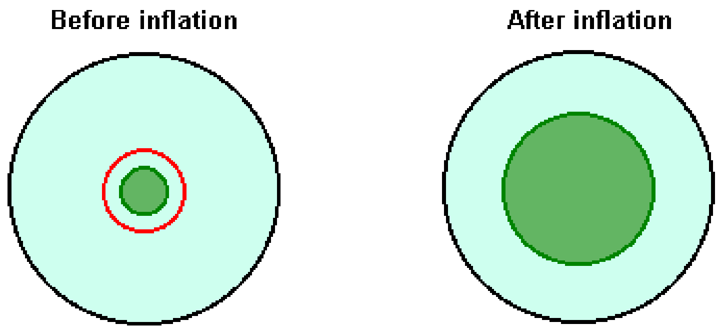
Negligible under normal circumstances (air pressure has a gravitational field 10^{-11} of the field generated by the air's mass density), but in the (very!!) early universe pressures were enormous – and mattered

Solving the equations of motion (we won't), we find that

$R(t) \sim \exp(Ct)$

i.e., the universe expands exponentially (with a time constant of of 10^{-34} or 10^{-33}s)

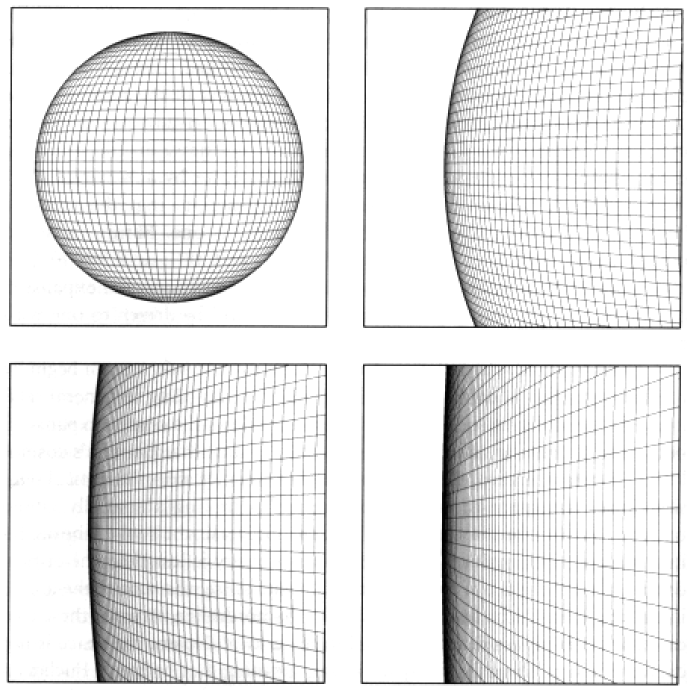




In green : volume filled with the matter inside the observable universe

In red : causal horizon at Planck's time

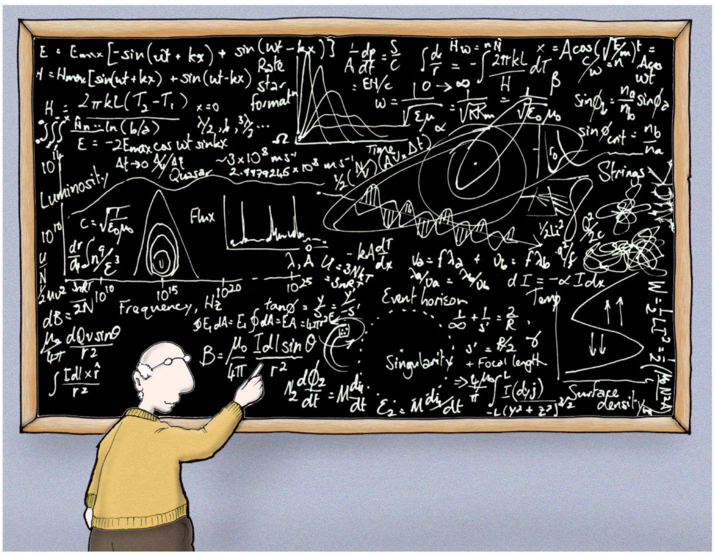
In black : today observable universe



Monopoles: inflated to negligible density

Structure: quantum fluctuations inflated to “galactic” scales

(+dark –matter cores).



Astrophysics made simple

(for now...)

The End